# Research on the allowable value of Lateral Breakthrough Error for Super long Tunnel from 20 to 50 kilometers

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**Abstract:** Up to now there is no specification about the allowable value of lateral breakthrough error for super long tunnel from 20 to 50 kilometers. Based on research on design of GPS network outside and traverse network inside tunnel we proposed a method to calculate the influence value caused by control surveying errors. Through a lot of simulative calculations and combined with piercing practice of super tunnels in Wan Jiazai Project province Shanxi we give out an allowable value table of lateral breakthrough error for super long tunnels from 20 to 50 kilometers finally, which is very important and valuable both for specification modifying and application.
**Key words:** super long tunnel; lateral breakthrough error; allowable value; simulative calculating; influence value

## 1 Preface

With the rapid development of economic construction in China, super long tunnels are becoming more and more, which are dug by the Tunnel driving Machine and have only one breakthrough profile, for instance the finished Qin Ling railway tunnel which is 18.4 km long, the 5# and 7# tunnel on the south main line of the finished Wan Jiazai Project Shanxi are 26.5 km and 42.6 km respectively, which leads the water of Yellow River from south to north of province Shanxi. All of them have only one breakthrough profile. Up to now there is no specification about the allowable value of lateral breakthrough error for super long tunnel from 20 to 50 km. Determining the allowable value of lateral breakthrough error for super long tunnel is an important problem that need to be studied and resolved.

We mean the breakthrough error as root mean square error (RMSE), the allowable value is 2 times of RMSE. For super long tunnel engineering it includes three errors in lateral, longitudinal, and vertical direction, in which the lateral breakthrough error is the most difficult one to meet the engineering requirements. So we focused the lateral breakthrough error in this paper.

The source of the lateral breakthrough error includes ground control surveying error outside tunnel, shaft contact surveying error, traverse surveying error inside tunnel and setting out error, among which setting out error can be neglected. There will be no shaft contact surveying error if there is only one breakthrough profile. If we use double-frequency GPS receivers to set up GPS network for the horizontal control outside tunnel, increase the GPS observation time and measurement segments, approach base line vectors with precision satellites almanac and special software etc., then man can decrease the lateral breakthrough error caused by GPS horizontal control surveying notably. The main influence for lateral breakthrough error in super long tunnel is traverse surveying error inside tunnel.

Since the tunnel size is limited, in general the diameter is 4-8 meter, the control network inside tunnel can only be a long and narrow traverse network. The angular error is the main error source for the lateral breakthrough error. In this paper we call the influence for lateral breakthrough error caused by control surveying errors as influence value, and proposed a method to calculate this value. Through a lot of simulative calculations and the practice in Wan Jiazai project province Shanxi we give out a table of allowable value of lateral breakthrough error for super long tunnels from 20 to 50 kilometers finally, which is very important and valuable both for specification modifying and application. At the same time the network design outside and inside tunnel is also discussed.

## 2 How to determine the allowable value of lateral breakthrough error

### *2.1 Design Size Method*

It is a method to determine the allowable value of lateral breakthrough error by Design Size of tunnel. Someone pointed out, that man can take the 1/7~1/10 of the diameter for circular tunnel and 1/7~1/10 of the width for square tunnel as the allowable value of lateral breakthrough error. It is easy to adjust tunnel axis near the breakthrough profile with this value and has no influence for tunnel quality, construction cost and rate. Take the Wan Jiazai Project for example, the diameter of 5# and 7# tunnel is 4.2 m, the 1/7 of diameter is 600 mm. As to railway tunnel, the diameter is about 6.0 m, its 1/10 is also 600 mm. If we take 600 mm as the allowable value of lateral breakthrough error for a tunnel, which is 50 km in length, then the RMSE of lateral breakthrough (half of the allowable value) is 300 mm. According to "equal influence" principle the influence value of traverse surveying inside tunnel is 245 mm, the allowable value is then 490 mm, the influence value of GPS network outside tunnel is 173 mm, and the allowable value is 346 mm. The value for tunnels shorter than 50 km can be appropriately decreased.

### *2.2 Weight Function Method*

#### *2.2.1 Calculating the influence value of GPS plane network outside tunnel*

The influence value of GPS plane network outside tunnel can be calculated with following weight function formula of breakthrough point for terrestrial control network [2] [3]:

$$
\begin{aligned}
d(\Delta y_p) = & -a_{ja}\,\Delta X_{jp}\,dx_j - (1 + b_{ja}\Delta X_{jp})\,dy_j + a_{ja}\,\Delta X_{jp}\,dx_a \\
& + b_{ja}\,\Delta x_{jp}\,dy_a - a_{cb}\,\Delta X_{cp}\,dx_b - b_{cb}\,\Delta X_{cp}\,dy_b \\
& + a_{cb}\,\Delta X_{cp}\,dx_c + (1 + b_{cb}\,\Delta X_{cp})\,dy_c \qquad (1)
\end{aligned}
$$

where $\Delta y_p$ is Y coordinate difference of breakthrough points calculated from entrance point and exportation point, respectively $\Delta X$ is X coordinate difference of corresponding points in tunnel coordinate system in which the X axis is from entrance point to exportation point, and the Y axis is in the direction perpendicular to X axis (see fig.1), a and b is coefficients which can be calculated by coordinate difference and distance corresponding points, index p is breakthrough point, index j is entrance point, index c is exportation point, index a is orientation point in entrance, index b is orientation point in exportation. This formula means that the influence value $M_{GPS}$ of GPS plane control network outside tunnel is related to the position and accuracy of entrance point, exportation point and corresponding orientation points, and also related to the position of the breakthrough point. We can calculate $M_{GPS}$ by covariance propagation rule, its allowable value $\Delta GPS$ is 2 $M_{GPS}$ . We call the method as coordinate difference weight function method, with which we should simulate a GPS network as a terrestrial network with all sides and all angles measured or as a network with all sides and all azimuths measured, and then calculate the simulating values $M_{GPS}$ with above formula and $\Delta GPS$. This method is proved to be correct and effective in theory and practice.

#### *2.2.2 Calculating the influence value of traverse network inside tunnel*

The traverse network inside tunnel is a very long and narrow network. There are two traverse networks from entrance and exportation point to the breakthrough profile respectively. The entrance-exportation point and its corresponding orientation points are known points obtained by the GPS plane control network outside tunnel. In fact, the influence value caused by the error of traverse network inside tunnel are the RMSE of Y coordinate of traverse points on the breakthrough profile relative to the known points at the entrance and exportation, and represented with $M_{JD}$ and $M_{CD}$ respectively. We can get it through adjustment of traverse network easily. If the breakthrough profile is at the middle of tunnel, then the influence values of the two traverse networks are equal, i.e. $M_{JD} = M_{CD} = M_D$.

### *2.2.3 Calculating the allowable value of the lateral breakthrough error*

From aforementioned calculating method for influence value inside and outside tunnel and the error propagation rule, we can get the lateral breakthrough error $M_{GT}$ as:

$$M_{GT} = \sqrt{M^2{}_{GPS} + M^2{}_{JD} + M^2{}_{CD}} = \sqrt{M^2{}_{GPS} + 2M^2{}_{D}} = \sqrt{M^2{}_{O} + M^2{}_{I}} \quad (2)$$

The allowable value of the lateral breakthrough error ΔGT is

$$\Delta_{GT} = 2M_{GT} = \sqrt{(2M_{GPS})^2 + (2\sqrt{2}M_D)^2} = \sqrt{\Delta^2{}_{O} + \Delta^2{}_{I}} \quad (3)$$

The allowable value of lateral breakthrough error caused by GPS control surveying error outside tunnel is:

$$\Delta GPS = 2M_{GPS} \quad (4)$$

The allowable value of lateral breakthrough error caused by traverse control surveying error inside tunnel is:

$$2\sqrt{2}M_D = 2\sqrt{2}M_{JD} = 2\sqrt{2}M_{CD} \quad (5)$$

For a super long tunnel from 20~50km in length, we can get $M_{GPS}$ and $M_D$ by simulating calculation. Through analysis and appropriate adjustment (refer to 4 and 5 in this paper), we can determine the allowable value of the lateral breakthrough error and its allowable value finally.

## 3 Design of control network outside and inside tunnel

### *3.1 Design of GPS plane control network outside tunnel*

A super long tunnel from 20 to 50 km can be dealt as straight-line network. The entrance point (J) and exportation point (C) are on tunnel center line, 3 orientation points (J1, J2, J3) and 3 orientation (C1, C2, C3) are at the entrance and exportation of the tunnel respectively (see fig.1). The orientation points should be at the place where it can be observed from the entrance point or exportation point. In order to decrease the effect of vertical deflection, the height difference between entrance or exportation points and orientation points should not be too big. We adopt an independent tunnel coordinate system in which the X axis is from entrance point (J) to exportation point (C), and the Y axis is in the direction perpendicular to X axis. The breakthrough profile is at the middle of the tunnel and parallel to Y axis. It is not necessary to set up any transition point even for a tunnel longer than 20 km. The difference between the longest side and the shortest one is very large, the longest side can be more than 50 km, the shortest one may be less than 300 m, because limited by topographical condition in mountains area and the requirements of inter-visibility. We should adopt enforced centering device to minimize the effect of sighting error and centering error for sides shorter than 300 m. For GPS network observation, we should use double frequency GPS receivers with accuracy not lower than 5mm+1ppm•D, and have enough independent base lines, for example: every point has at least 4 independent base lines in different observation segment, the long base line number between the entrance and exportation should not be less than 5, the observation time of long base lines should be much longer, and use precision satellites almanac to approach long base line vectors.

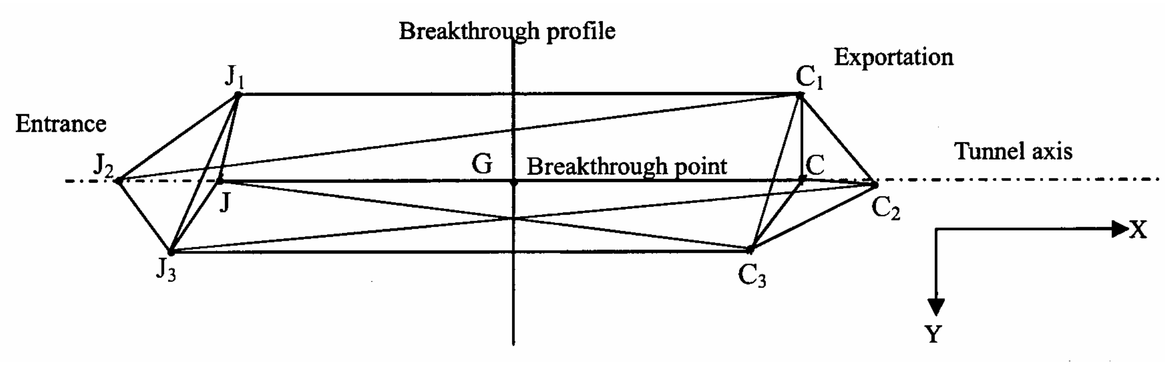
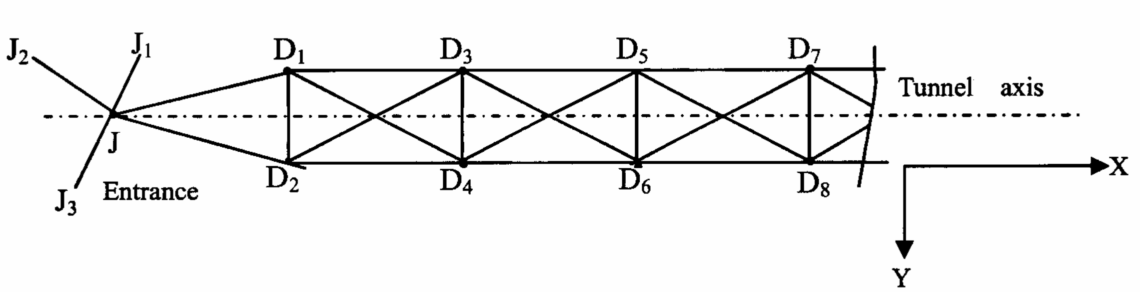


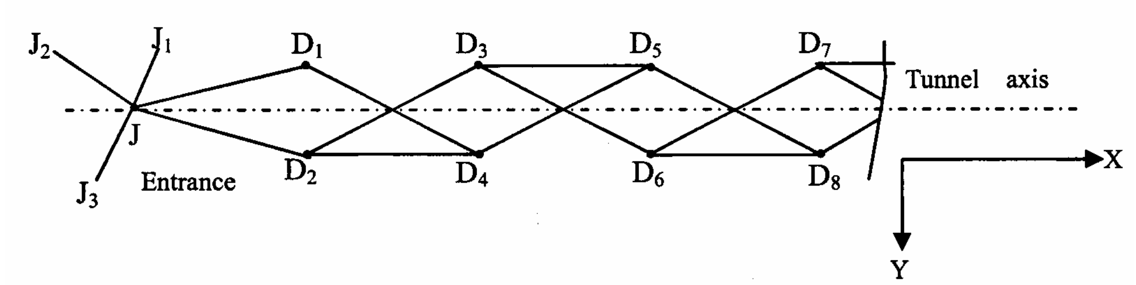

*Fig.1 Design of GPS network outside the super long tunnel*

### *3.2 Design of traverse network inside tunnel*

We set up traverse network inside tunnel based on GPS plane control network. The traverse network can be designed with two categories: full traverse network constructed with braced quadrilateral and double traverse network, shown as figure 2(a) and (b).




*(a) Full traverse network*




*(b) Double traverse network*

*Fig.2 Design of traverse network inside tunnel*

The length of the long sides of traverse is designed as 500 m. Because the observation number of full traverse network (see figure 2(a)) is very large, and the sides close to the tunnel wall are readily effected by lateral refract. We should choose the double traverse network, and have a checking every two lateral sides (see figure 2(b)). We should adopt electronic total stations with accuracy better than 1″ (angle ) and 3mm+2ppm•D (distance ) for the observation of traverse network inside tunnel, the two short sides of the braced quadrilateral can be measured with metal band, and the direction on short side need not to measure.

## 4 Simulating calculation

### *4.1 GPS network outside tunnel*

For the GPS network outside tunnel shown in figure 1, we simulate the network as one with all the sides and all the directions measured, in which the angle accuracy is 0.7″, and distance accuracy is 5mm+1ppm•D, and we simulate the network also as another one with all the sides and all the azimuths measured, in which the angle accuracy is 1.0″ and the distance accuracy is the same as former. We calculate the influence value of the lateral breakthrough error for super long tunnel from 20 to 50 km respectively, there is no significant distinguish for above two sorts, the results are shown in table 1.

| Tunnel length (km) / Type | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|---|
| Calculated Influence Value of Lateral Breakthrough Error | 47~55 | 58~68 | 70~82 | 82~95 | 94~108 | 106~122 | 117~137 |
| Suggested Influence Value | 51 | 63 | 76 | 88 | 101 | 114 | 127 |

*table 1: influence value (mm) of GPS network outside tunnel from 20 to 50 km*

### *4.2 Traverse network inside tunnel*

We simulate the networks in figure 2 (a) and (b) with the angle accuracy 0.7″ and distance accuracy 1mm+1ppm•D. The results for full traverse network and double traverse network inside tunnel with 20 km are listed in table 2.

| Items | Full traverse network | Double traverse network |
|---|---|---|
| Number of Observations | 165 | 95 |
| Number of Linear Observations | 103 | 48 |
| Redundancy | 141 | 19 |
| Accuracy of Weakest Point | 78.5 (mm) | 95.0 (mm) |

*table 2: compare of two traverse network inside super long tunnel with 20 km*

The influence values of the lateral breakthrough error of full traverse network and double traverse network for tunnel which is 4~20 km in length are listed in table 3 for comparison.

| Tunnel Length (km) | Full traverse network | Double traverse network | Difference |
|---|---|---|---|
| 4.0 | 11.8 | 13.0 | 1.2 |
| 8.0 | 25.3 | 29.5 | 4.2 |
| 10.0 | 33.0 | 39.0 | 6.0 |
| 12.0 | 41.0 | 49.3 | 8.3 |
| 15.0 | 54.2 | 66.2 | 12.0 |
| 16.0 | 58.9 | 72.2 | 13.3 |
| 20.0 | 78.5 | 95.0 | 16.5 |

*table3: compare of influence values of two traverse networks inside tunnel from 4 to 20 km*

The influence values of the lateral breakthrough error of full traverse network and double traverse network inside tunnel from 20~50 km with different angle and distance accuracy are listed in table 4.

| Tunnel length (km) / Accuracy | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|---|
| 0.7”, 1 mm + 1 ppm | 95 | 130 | 166 | 208 | 250 | 300 | 344 |
| 0.4”, 1 mm + 1 ppm | 55 | 74 | 96 | 120 | 145 | 173 | 200 |
| 0.4”, 3 mm + 2 ppm | 56 | 75 | 100 | 124 | 150 | 178 | 206 |

*table 4: influence values (mm) of double traverse networks inside tunnel from 20 to 50 km*

### *4.3 Results analysis*

From table 1, 2, 3 and 4 we can make following analysis:

(1) The influence value of GPS network outside tunnel is nearly proportion to the tunnel length. Because the maximum of influence value is not very big, the increment magnitude with the tunnel length is also not significant.

(2) The difference of influence value between full traverse network and double traverse network is not significant, but the observation amount and cost of full traverse network is much more. We should choose therefore the double traverse network outside tunnel.

(3) For a super long tunnel with same length, the influence value of traverse network inside tunnel is bigger than that of GPS network outside tunnel notably.

(4) The influence value of traverse network inside tunnel is changed proportion to angle accuracy.

(5) The influence value for longitudinal breakthrough error of network inside and outside tunnel are very small relatively. We have calculated but not listed it in this paper.
(6) The influence values for lateral breakthrough error determined with Design Size Method have significant difference comparing to the results of our method proposed in this paper. Design Size Method is quite simple but not reasonable.

### *4.4 Analysis of breakthrough error in practical project*

The sum Length of tunnel group in main line and south main line of Wan Jiazai Project province Shanxi is 212.6 km, the 5# and 7# tunnel on south main line are 26.5 km and 42.6 km respectively. The biggest breakthrough error is only 85 mm. We have got breakthrough errors of 18 tunnels, the values are in the range of 9 mm to 85 mm, and only 6 value exceed 40 mm. The control networks inside tunnel of Wan Jiazai Project were double traverse network. The practice proved that it is realizable for a correct breakthrough of super long tunnel with very high precision.

## 5 Determination and allotment of allowable value of the lateral breakthrough error for super long tunnel

From simulating calculation and analysis, we can get influence values and allowable value of lateral breakthrough error for super long tunnel from 20 to 50km, they are listed in table 5. We make some Adjusting for the values in table 5, and get the allowable values of lateral breakthrough error for super long tunnel (listed in table 6). The values in table 6 have significant difference with the values of tunnel from 4~20km with one breakthrough profile in the existing norm (see table 7). The value of this paper is more strict and its allotment is more reasonable.

| Tunnel length (km) / Type | 17~20 | 20~25 | 25~30 | 30~35 | 35~40 | 40~45 | 45~50 |
|---|---|---|---|---|---|---|---|
| Influence values of GPS network | 51 | 63 | 76 | 88 | 101 | 114 | 127 |
| Influence values of traverse network from Entrance | 95 | 130 | 166 | 208 | 250 | 300 | 344 |
| Influence values of traverse network from Exportation | 95 | 130 | 166 | 208 | 250 | 300 | 344 |
| Sum of influence values | 144 | 195 | 247 | 307 | 368 | 440 | 503 |
| Allowable values | 288 | 390 | 494 | 614 | 736 | 880 | 1006 |

*table 5: influence values and allowable values (mm) of lateral breakthrough error for tunnel from 20 to 50 km*

| Tunnel length (km) / Type | 17~20 | 20~25 | 25~30 | 30~35 | 35~40 | 40~45 | 45~50 |
|---|---|---|---|---|---|---|---|
| Outside GPS network | 100 | 130 | 160 | 180 | 200 | 230 | 250 |
| Inside Traverse network | 282 | 378 | 474 | 592 | 712 | 850 | 968 |
| Sum | 300 | 400 | 500 | 620 | 740 | 880 | 1000 |

*table 6: the allowable values (mm) of lateral breakthrough error of tunnel from 20 to 50 km*

| | 4~8 | 8~10 | 10~13 | 13~17 | 17~20 |
|---|---|---|---|---|---|
| For network outside tunnel | 90 | 120 | 180 | 240 | 300 |
| Traverse network inside tunnel | 120 | 160 | 240 | 320 | 400 |
| Sum | 150 | 200 | 300 | 400 | 500 |

*table 7: the allowable values (mm) of lateral breakthrough error of tunnel from 4 to 20 km in norm*

## 6 Conclusion

Determination and allotment of the allowable value of lateral breakthrough error for super long tunnel from 20 to 50 km is a very important problem need to solve urgently. This paper proposed a method and a set of solution. It is significant both for research and practice. With modern measurement technique the error of GPS network outside tunnel has less effect on the lateral breakthrough error, and the main influence is angle observation error of traverse network inside tunnel, it is better to set up network inside tunnel with double traverse network. The lateral breakthrough error could not be allotted according to "equal influence principle". Compare with lateral breakthrough error the longitudinal and vertical breakthrough errors are relative small and easy to meet the design requirement.

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